

Basic ideas in consumer theory

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Introduction

- ▶ Consumer theory is most basic and widely used tool in the economists' tool-set'
- ▶ Two basic questions:
 1. What will consumer choose at different prices & income?
 2. What is the impact of a change in prices on consumer's well-being?

Outline

1. Basic choice theory
2. Neoclassical economics' use of choice theory
3. Consumer demand
4. Consumer welfare: basic idea
5. Consumer welfare: measuring the effect of a price change

I

Basic choice theory

Basic choice theory: Introduction

- ▶ Three alternative views of how to think about modeling individual choice:
 1. Utility maximization approach
 2. Preference maximization approach
 3. Revealed preference approach

Basic choice theory: Utility maximization

- ▶ X – set of all conceivable choice items for individual
- ▶ $B \subseteq X$ – budget set describing available items in a particular choice situation
- ▶ Basic assumptions:
 1. Subjects have a utility function $u(x)$ that assigns a value to every choice alternative
 2. Subjects always make choice that maximizes utility function over budget set
- ▶ Consumer demand:

$$x^*(B) = \operatorname{argmax}_{x \in B} u(x)$$

Basic choice theory: Utility maximization

► Notes:

- This model based on literal assumption that there is a utility function that is maximized to make choices
- Model has several implicit assumptions:
 1. Subject values and compares all options at the time of choice
 2. Valuation independent of budget set and other properties of choice situations (e.g., $u(a)$ not affected by which other items are present in the budget set

Basic choice theory: Preference maximization

- ▶ \succsim - preference relation over options in choice set
- ▶ $x \succsim y$ means x is at least as good as y
- ▶ A preference relationship is:
 - ▶ complete: if for all x, y we have $x \succsim y$ or $y \succsim x$ (or both)
 - ▶ transitive: if for all x, y, z we have that $x \succsim z$ whenever $x \succsim y$ and $y \succsim z$
- ▶ Rational preferences are complete and transitive
- ▶ Intuition:
 - ▶ Rational preferences lead to full orderings of X
 - x
 - ▶ Ex: y^z
 - w
 - ...

Basic choice theory: Preference maximization

- ▶ (Weak) preference relationship \succsim primitive of theory
- ▶ Using \succsim can define:
 - ▶ Strict preference relation \succ
 - ▶ Indifference relationship \sim
- ▶ Basic assumptions:
 1. Subjects have a rational preference relation that (weakly) ranks all possible alternatives
 2. Subjects always chose option(s) ranked higher within the budget set
- ▶ Consumer demand:

$$x * (B) = \{x \in B \mid x \succsim y \text{ for all } y \in B\}$$

Basic choice theory: Preference maximization

► Note:

- This model makes slighter weaker assumptions than utility maximization
- Basic assumptions:
 1. Subjects are able to compare any two alternatives and rank them (weakly) in terms of desirability
 2. When presented with a budget set, they can apply the comparison process easily to select highest ranked alternative in set
- Why weaker?
 - No assumption that alternatives get assigned “values/utility” to make choices
 - Instead subjects only assumed to be able to tell which option in a pair is better

Basic choice theory: Connection between UM and PM

- ▶ A preference relation \succsim is represented by a utility function u iff $u(x) \geq u(y)$ whenever $x \succsim y$.
- ▶ As long as some technical conditions are satisfied, most rational preference relationships can be represented by many utility functions
- ▶ Why many?

x	3	10000000	
y	2	1	...
z	1	-27	
- ▶ If u represents \succsim , so does every strictly increasing transformation of u

Basic choice theory: Connection between UM and PM

- ▶ IMPORTANT:

- ▶ Preference relations only contain ordinal information about:
 - ▶ They allow to determine if x preferred to y or z preferred to w
 - ▶ But have no info about whether x preferred to y by more/less than z is preferred to w

Basic choice theory: Revealed preference

- ▶ Motivation for alternative model:
 - ▶ Before neuroeconomics, widely accepted in economics that cannot measure utility or preference directly
 - ▶ This is troublesome for economists for two reasons:
 1. It means that key assumption in the theory cannot be tested directly
 2. Since utility and preferences drive choices, how can predictions be made without knowing them?
- ▶ Solution to this problem is the theory of revealed preference
- ▶ This theory is the way modern economists are trained to think about modeling individual behavior

Basic choice theory: Revealed preference

- ▶ The primitive of the theory is observed choice data in the form of a choice function $C(B_k)$ that describes the choice made by the individual in various choice sets B_k
- ▶ From this choice data possible to construct a mathematical construct called a revealed preference relation \succsim^R
- ▶ How?
 - ▶ Define $x \succsim^R y$ whenever there is a budget set B in the observed data with $x, y \in B$ and $x \in x^*(B)$
- ▶ Intuition:
 - ▶ If x is chosen when y is available then individual “reveals” that x is at least as good and y

Basic choice theory: Revealed preference

- ▶ Important:
 - ▶ x revealed preferred to y does not mean that we have learned that the brain is assigning a higher utility or preference to x
 - ▶ All that we have learned is that there is a choice situation in which, through whatever processes the brain uses to make choices, it chooses x over y
- ▶ Thus, revealed preference
 - ▶ IS a summary of choice behavior
 - ▶ IS NOT a measure of relative utility or preference

Basic choice theory: Connection between PM and RP

- ▶ Basic question: Are there conditions under which the observed choice data is COMPATIBLE with the maximization of an underlying preference function?
- ▶ A choice function C satisfies the Weak Axiom of Revealed Preference (WARP) iff we don't have budget sets B, B' and options x, y that belong to both of them such that:
 - ▶ $x \in C(B)$ and $y \notin C(B)$
 - ▶ But $y \in C(B')$ and $x \notin C(B')$
- ▶ Idea:
 - ▶ $x \in C(B)$ and $y \notin C(B)$ implies that $x \succ^R y$
 - ▶ $y \in C(B')$ and $x \notin C(B')$ implies that $y \succ^R x$
 - ▶ This reveals an underlying inconsistent preferences

Basic choice theory: Revealed preference

Theorem

- 1. If the observed data C satisfies WARP then there exists a rational data that, when maximized, generates behavior consistent with C .*
- 2. If C includes data about choices over all possible pairs and triples of options, then the revealed preference relation is the only relation that rationalizes the observed data*

Basic choice theory: Revealed preferences

► Intuition 1:

- Suppose that C satisfies WARP, but there is sparse choice data

- $C(\{x, y, z\}) = x$ and $C(\{y, z, w\}) = y$

$$C(\{x, y, z\}) = x \quad C(\{y, z, w\}) = y$$

- Then:

$$\begin{array}{cc} x & y \\ yz & zw \end{array}$$

- Many possible preferences consistent with C :

$$\begin{array}{ccc} x & w & x \\ y & x & w \\ z & y & y \quad \dots \\ w & x & z \end{array}$$

Basic choice theory: Revealed preferences

► Intuition 2:

- Suppose that C satisfies WARP and very complete

- $C(\{x, y, z\}) = x$ and $C(\{y, z\}) = y$

$$C(\{x, y, z\}) = x \quad C(\{y, z\}) = y$$

- Then:

$$\begin{array}{ccc} x & & \\ yz & & y \\ & & z \end{array}$$

- Then unique preferences consistent with C :

$$\begin{array}{c} x \\ y \\ z \end{array}$$

Basic choice theory: Revealed preferences

▶ Intuition 3:

- ▶ Suppose that C violates WARP

- ▶ $C(\{x, y\}) = x$, $C(\{y, z\}) = y$ and $C(\{x, z\}) = z$

$$C(\{x, y\}) = x \quad C(\{y, z\}) = y \quad C(\{x, z\}) = z$$

- ▶ Then:

x	z
y	x
	z

- ▶ Here there is no rational ordering consistent with C

II Neoclassical Economics Use of Choice Theory

Neoclassical Econ: Worldview

- ▶ Economists extremely skeptical that we can measure preferences & utility functions with any precision that would be useful for economics
- ▶ Some, like Gul and Psendorfer, assume that only valid source of data is behavior
- ▶ As a result, economists view choices as the primitive input to economic models of consumer behavior
- ▶ Logis of the intellectual exercise:
 - ▶ Observe consumer choices in an initial set of situations (summarized by C)
 - ▶ Infer restrictions on underlying revealed preferences from this
 - ▶ Use these revealed preferences to make predictions about novel choice situations

Neoclassical economics: Example

- ▶ Initial behavioral data: $C(\{x, y, z\}) = x$ and $C(\{y, z, w\}) = y$
- ▶ Inferences about revealed preference: $\frac{x}{yz}$ and $\frac{y}{zw}$
- ▶ Revealed preference must satisfy: $\frac{x}{y}$, $\frac{y}{zw}$, but no restrictions otherwise
- ▶ Predictions:
 - ▶ $C(\{x, z, w\}) = x$
 - ▶ $C(\{zw\})?$
- ▶ Key lesson: The more data is available, the more predictions can be made!

Neoclassical economics: Notes

1. Key ideas consistency!

- ▶ Whatever processes are at work they are supposed to be “consistent” across situations and time, in the sense of being consistent with WARP

2. Isn't this tautological?

- ▶ No. In most practical applications we have only a small amount of data, so we make out-of-sample predictions

Neoclassical economics: Limitation 1

- ▶ Problem: In practice have too few data to be able to sufficiently restrict γ^R
 - ▶ Standard solution:
 1. Impose functional form restrictions (so only have to estimate a few parameters)
 2. Pool data across subjects
 - ▶ Both methods are a violation of strict neoclassical logic, but it is unavoidable

Neoclassical economics: Limitation 2

- ▶ Problem: Theory assumes no stochastic/noisy choice, which is false in practice
 - ▶ Standard solution:
 - ▶ Random Utility Models

Neoclassical economics: Limitation 3

- ▶ Problem: Data in practice can exhibit sizable violations of WARP
 - ▶ Standard solution:
 - ▶ Extend theory by assuming that consistency only holds within similar types of “choice situations”
 - ▶ s – choice situation
 - ▶ Data $C(B, s)$
 - ▶ Infer choice situation dependent revealed preferences $\succsim^R(s)$
 - ▶ Conceptual problem:
 - ▶ Without further restrictions the theory becomes unfalsifiable
 - ▶ Any time that there is a violation of WARP just define a new s

III

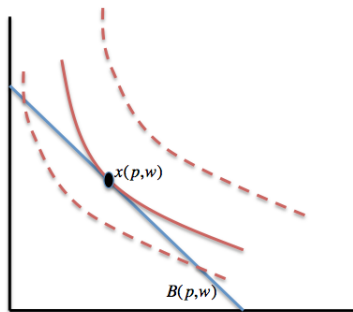
Consumer theory

Consumer theory: Basic question

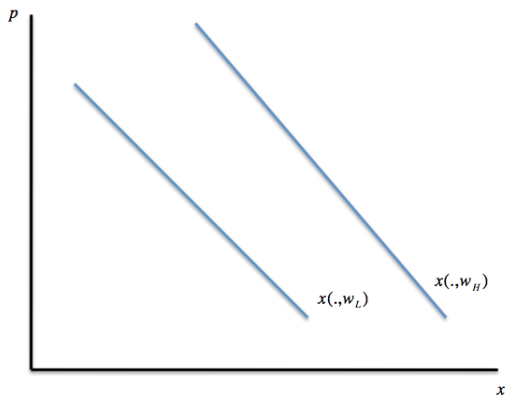
- ▶ Consumption bundles: $x = (x_1, \dots, x_n)$
- ▶ Prices: $p = (p_1, \dots, p_n)$
- ▶ Wealth: w
- ▶ $B(p, w)$
- ▶ Assumption: Consumers have an exogenous given preference relation represented by u
- ▶ Basic question: What bundle $x(p, w)$ should the consumer buy?
- ▶ Key assumptions:
 - ▶ Price-taking
 - ▶ Linear pricing
 - ▶ Perfectly divisible goods

Consumer theory: Marshallian demand

- ▶ $x(p, w) = \operatorname{argmax}_{x \in B(p, w)} u(x)$
- ▶ Key property of optimal consumption: $MRS_{j,k} = \frac{\partial u / \partial x_j}{\partial u / \partial x_k} = \frac{p_j}{p_k}$



Consumer theory: Demand curves



Consumer theory: Quasilinear-preferences

- ▶ $u(x_1, \dots, x_n, m) = f(x_1, \dots, x_n) + m$
- ▶ Why useful?
 - ▶ $x(p, w) = \phi(p)$
 - ▶ i.e., demand for goods 1 to n is independent of m

Consumer theory: Basic properties of Marhsallian demand

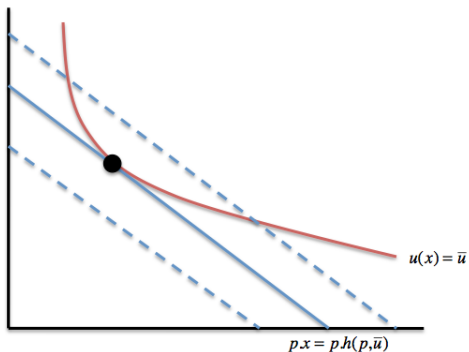
- ▶ $x(p, w) = x(\lambda p, \lambda w)$ for all $\lambda > 0$
- ▶ Walras law: $\sum p_k x_k(p, w) = w$ whenever u is strictly increasing
- ▶ $\frac{\partial x_k}{\partial w} > 0$?
 - ▶ Depends of the utility function and level of wealth
 - ▶ True for “normal goods”, false for “inferior goods”
- ▶ $\frac{\partial x_k}{\partial p_k} < 0$?
 - ▶ Depends of the utility function and level of wealth
 - ▶ Regular good if < 0 , Giffen good if > 0

Consumer theory: More on Marshallian demand

- ▶ Indirect utility function: $v(p, w) = u(x(p, w))$
- ▶ Walrasian demand is the most common object used in economics
- ▶ Examples of use:
 - ▶ savings (x_t denotes consumption in different dates, w is lifetime wealth)
 - ▶ charitable giving ($u(x, g)$, with x private consumption, g charitable donation)

Consumer theory: Hicksian or compensated demand

- ▶ $h(p, \bar{u}) = \operatorname{argmin}_{u(x) \geq \bar{u}} p \cdot x$
- ▶ $e(p, \bar{u})$: minimized expenditure

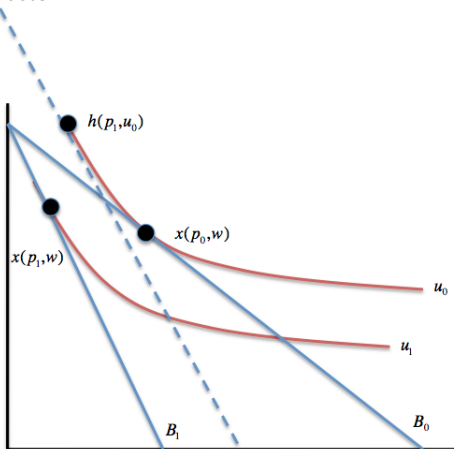


Consumer theory: Properties Hicksian demand

- ▶ $\frac{\partial h_k}{\partial \bar{u}} > 0$
- ▶ Law of demand: $(p' - p)(h(p', u) - h(p, u)) \leq 0$
- ▶ Especial case of law of demand: $\frac{\partial h_i}{\partial p_i} \leq 0$
- ▶ $\frac{\partial e}{\partial p_k} = h_k(p, \bar{u})$

Consumer theory: Slutsky equation

- ▶ $\frac{\partial x_i(p,w)}{\partial p_j} = \frac{\partial h_i(p,\bar{u})}{\partial p_j} - \frac{\partial x_i(p,w)}{\partial w} x_j(p,w)$
- ▶ Why useful?
 - ▶ Decompose price changes into substitution and income effects



IV

Basic Welfare Economics

Welfare economics: Basic Idea

- ▶ LIBERTARIAN PRINCIPLE: outcome x is better than outcome y iff the subject would have chosen x over y by themselves
- ▶ This leads to an ordinal welfare ranking over alternatives

Welfare economics: Discussion

- ▶ In order to make welfare comparisons, economists need to know how consumers would choose
- ▶ Soft-version of view:
 - ▶ Individuals always maximize utility
 - ▶ We can't measure their utility directly
 - ▶ Nevertheless, the methodology still gets the utility comparisons correctly
- ▶ Hard-version of view:
 - ▶ Respect people choices even if they may mistakes
 - ▶ Very skeptical can tell when mistakes are being made

V

Welfare economics: Price changes

Welfare price changes: Problem definition

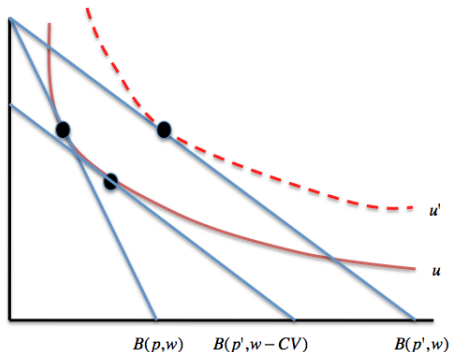
- ▶ Basic question: What is the change in consumer welfare of changing prices from p to p'
- ▶ Answer: $v(p', w) - v(p, w)$
- ▶ Careful w/ interpretation: there is only ordinal information in utility function
- ▶ Tricky to implement since indirect utility function unknown
- ▶ Fortunately there is an alternative solution

Welfare price changes: Alternative definition

- ▶ Alternative question: How much money is required to achieve the same utility before and after the price change?
- ▶ Two different versions of measure:
 - ▶ Compensating variation (CV): compare at initial utility
 - ▶ Equivalent variation (EV): compare at new utility

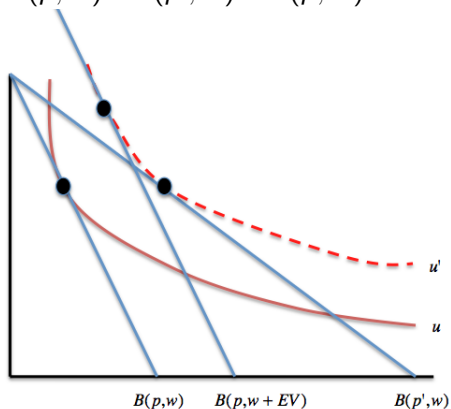
Welfare price changes: CV

- ▶ CV measures how much less wealth will the consumer need to achieve the same utility at prices p' that she achieved at p
- ▶ $u = v(p, w)$
- ▶ $CV = e(p, u) - e(p', u) = w - e(p', u)$



Welfare price changes: EV

- ▶ EV measures how much money would the consumer have needed at prices p to achieve the same utility that she achieved at p'
- ▶ $EV > 0$ iff better off
- ▶ $u' = v(p', w)$
- ▶ $EV = e(p, u') - e(p', u') = e(p, u') - w$

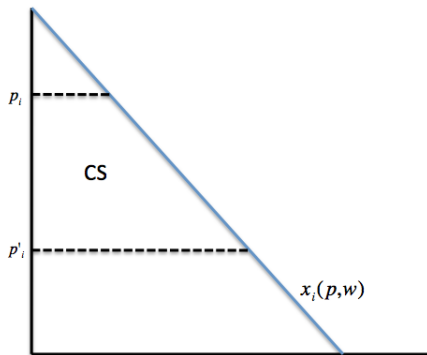


Welfare price changes: CV vs EV

- ▶ If the price change only affects a single good we have that:
 - ▶ $CV = e(p, u) - e(p', u) = \int_{p'_i}^{p_i} h_i(p, u) dp_i$
 - ▶ $EV = e(p, u') - e(p', u') = \int_{p'_i}^{p_i} h_i(p, u') dp_i$
- ▶ Generally CV and EV not the same
- ▶ They coincide for quasilinear-utility (because there Hicksian demand is independent of u)

Welfare price changes: Consumer surplus

- ▶ Consumer surplus is commonly used in empirical work
- ▶ $CS = \int_{p_i}^{p_i'} x_i(p, w) dp_i$
- ▶ Not an exact measure of welfare
- ▶ However, if good normal or inferior, then $\min\{CV, EV\} \leq CS \leq \max\{CV, EV\}$



Welfare price changes: Multiple subjects

- ▶ Typical practice: Sum the individual EV, CV, or CS
- ▶ Implicitly assumes no distributional preferences

VI

Final discussion

Want to learn more?

1. Microeconomic Theory (Mas-Colell, Whinston and Greene)
2. Intermediate Microeconomics (Hal Varian)

Welfare economics w/ Irrational consumers?

- ▶ Bernheim and Rangel, 2009, Quarterly Journal of Economics, “Behind Revealed Preference: Choice Theoretic Foundations for Behavioral Welfare Economics”